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Abstracts Booklet

Organizers: Maria Axenovich, Yury Person and Dieter Rautenbach.

Local organizers: Mónika Csikós, Jonathan Rollin, Torsten Ueckerdt and Yelena Yuditsky.
Counting in hypergraphs: from small to large

Peter Allen
London School of Economics and Political Science
London, UK

Abstract

We show that a fairly weak notion of pseudorandomness in sparse hypergraphs, in terms of counting small subgraphs, is enough to count vertex-by-vertex embeddings of all large sparse hypergraphs. I will state the result, very briefly sketch the proof and claim that it applies in the sparse hypergraph regularity setting.

This is joint work with Ewan Davies and Jozef Skokan.
Extremal and near extremal Ryser hypergraphs
Anurag Bishnoi
Freie Universität Berlin
Berlin, Germany

Abstract

Ryser’s conjecture states that every $r$-uniform $r$-partite hypergraph $\mathcal{H}$ satisfies $\tau(\mathcal{H}) \leq (r - 1)\nu(\mathcal{H})$. This is only known to be true for $r = 2$, where it is König’s theorem from graph theory, and for $r = 3$ where Aharoni gave a topological proof. Beyond this, there are some special cases of the conjecture that have been proved assuming other restrictions on the hypergraph. In recent years, there have been some interest in studying the extremal examples of this conjecture, that is, hypergraphs $\mathcal{H}$ which satisfy $\tau(\mathcal{H}) = (r - 1)\nu(\mathcal{H})$. These hypergraphs are called Ryser hypergraphs. For $\nu = 1$, the only known infinite families of Ryser hypergraphs are for $r = q + 1$ and $r = q + 2$ where $q$ is a prime power, and both of these constructions involve finite projective planes. For $\nu > 1$, one can take disjoint copies of intersecting Ryser hypergraphs, and in fact it was been proved that for $r = 3$ this is essentially the only way to get Ryser hypergraphs of higher matching number. In this talk we will see that such a characterisation fails for higher values of $r$ by giving a new family of non-intersecting Ryser hypergraphs with $r = q + 1$, $q$ a prime power greater than 4, whose vertex set cannot be partitioned into intersecting Ryser subhypergraphs. I will also present some near extremal constructions of intersecting hypergraphs from finite geometries which can potentially be useful in constructing new families of Ryser hypergraphs.

Joint work with Valentina Pepe.
Perfectly packing degenerate graphs with many leaves
Julia Böttcher
London School of Economics and Political Science
London, UK

Abstract

A perfect packing of a family \((G_i)_{i \in [m]}\) of graphs in a host graph \(H\) is a colouring of the edges of \(H\) with colours \([m]\) such that the edges with colour \(i\) form a copy of \(G_i\) for each \(i \in [m]\). We show that for large \(n\) we can perfectly pack any family \((G_i)_{i \in [m]}\) of \(d\)-degenerate graphs with maximum degree at most \(cn/\log n\) into \(K_n\) if \(\sum e(G_i) = n(n - 1)/2\) and if for \(i \geq m - \eta n\) we have \(v(G_i) \leq (1 - \eta)n\) and that \(G_i\) has at least \(\eta n\) leaves.

This is joint work with Peter Allen, Dennis Clemens, Anusch Taraz.

Distributed coloring in planar graphs
Marthe Bonamy
CNRS, LaBRI, Université de Bordeaux
Bordeaux, France

Abstract

We are concerned with efficiently coloring sparse graphs in the distributed setting with as few colors as possible. According to the celebrated Four Color Theorem, planar graphs can be colored with at most 4 colors, and the proof gives a (sequential) quadratic algorithm finding such a coloring. A natural problem is to improve this complexity in the distributed setting. Using the fact that planar graphs contain linearly many vertices of degree at most 6, Goldberg, Plotkin, and Shannon obtained a deterministic distributed algorithm coloring \(n\)-vertex planar graphs with 7 colors in \(O(\log n)\) rounds. Here, we show how to color planar graphs with 6 colors in polylog \(n\) rounds. Our algorithm indeed works more generally in the list-coloring setting and for sparse graphs (for such graphs we improve by at least one the number of colors resulting from an efficient algorithm of Barenboim and Elkin, at the expense of a slightly worst complexity). Our bounds on the number of colors turn out to be quite sharp in general. Among other results, we show that no distributed algorithm can color every \(n\)-vertex planar graph with 4 colors in \(o(n)\) rounds. This is joint work with Pierre Aboulker, Nicolas Bousquet and Louis Esperet.
The edge version of the Erdős-Pósa property
Henning Bruhn(-Fujimoto)
Ulm University
Ulm, Germany

Abstract

For many graph classes $C$ it is known whether they possess the Erdős-Pósa property, that is, whether there is a function $f$ such that every graph $G$ that doesn’t contain $k$ disjoint members of $C$ admits a vertex set $X$ of size $|X| \leq f(k)$ that meets every member of $C$ in $G$. The prime example is the class of cycles: that it has the property is the content of the classic theorem of Erdős and Pósa that started this line of research.

The edge version of the Erdős-Pósa property seems equally natural: here, we seek edge-disjoint members of $C$, or alternatively, settle for an edge set $X$ that meets all members of $C$. Strikingly, the edge version appears quite a bit more mysterious than the ordinary, vertex version. While a good number of graph classes have both properties, or both have none, some have one but not the other. In the talk, I will try, but in some sense fail, to shed some light on this phenomenon. The walk will be based on joint work with Matthias Heinlein, Felix Joos and Arthur Ulmer.

Recent results on extremal numbers
David Conlon
University of Oxford
Oxford, UK

Abstract

Given a graph $H$ and a natural number $n$, the extremal number $\text{ex}(n,H)$ is the largest number of edges in an $H$-free graph on $n$ vertices. This function is reasonably well understood when $H$ has chromatic number at least 3 but the bipartite case remains wide open. In this talk, I will describe some recent work on this challenging question.
Induced saturation of graphs
Mónika Csikós
Karlsruhe Institute of Technology
Karlsruhe, Germany

Abstract

A graph $G$ is $H$-saturated for a graph $H$, if $G$ does not contain a copy of $H$ but adding any new edge to $G$ results in such a copy. An $H$-saturated graph on a given number of vertices always exists and the properties of such graphs, for example their highest and lowest density, have been studied intensively.

A graph $G$ is $H$-induced-saturated if $G$ does not have an induced subgraph isomorphic to $H$, but adding an edge to $G$ from its complement or deleting an edge from $G$ results in an induced copy of $H$. It is not immediate anymore that $H$-induced-saturated graphs exist. In fact, Martin and Smith [?] showed that there is no $P_4$-induced-saturated graph. Behrens et al. [?] proved that if $H$ belongs to a few simple classes, then there is an $H$-induced-saturated graph.

This talk addresses the existence question for $H$-induced-saturated graphs. We present some classes of graphs $H$ such that there is a Hamming graph which is $H$-induced-saturated. These classes include large families of trees and special subdivisions of certain graphs with a unique maximum degree. Furthermore, we give a complete characterization of all connected graphs $H$ for which there exists an $H$-induced-saturated Hamming graph of dimension 2.

This is joint work with Maria Axenovich.

References


Perturbed Ramsey problems
Shagnik Das
Freie Universität Berlin
Berlin, Germany

Abstract
We say a graph $G$ is Ramsey for a pair of graphs $(H_1,H_2)$ if any red-/blue-colouring of $E(G)$ contains either a red $H_1$ or a blue $H_2$. Much research has been devoted to determining the thresholds at which the Erdős–Rényi random graph $G(n,p)$ becomes Ramsey for $(H_1,H_2)$. In this talk, we instead consider the perturbed random graph model, where one sprinkles some random edges on top of a deterministic dense graph, and ask how much less randomness is needed to ensure the resulting graph is Ramsey.

This line of research was initiated by Krivelevich, Sudakov and Tetali in 2006, who determined the thresholds for the pairs $(K_3,K_t)$, $t \geq 3$. Here we discuss perturbed Ramsey thresholds for cycles vs cycles, cycles vs cliques, and cliques vs cliques.

This is joint work with Andrew Treglown.

(a : b)-choosability
Zdeněk Dvořák
Charles University
Prague, Czech Republic

Abstract
In 1980, Erdős, Rubin and Taylor asked whether for all positive integers $a$, $b$, and $m$, every $(a : b)$-choosable graph is also $(am : bm)$-choosable. We will survey results motivated by this conjecture and provide a negative answer by exhibiting a 4-choosable graph that is not $(8 : 2)$-choosable.

The talk is based on a joint result with Xiaolan Hu and Jean-Sébastien Sereni.
Dynamic monopolies and degenerate sets
Stefan Ehard
Ulm University
Ulm, Germany

Abstract
A widely studied model for influence diffusion in social networks are dynamic monopolies. For a graph $G$ and an integer-valued threshold function $\tau$ on its vertex set, a dynamic monopoly is a set of vertices of $G$ such that iteratively adding to it vertices $u$ of $G$ that have at least $\tau(u)$ neighbors in it eventually yields the entire vertex set of $G$. In this talk we present recent bounds, algorithms, and hardness results for minimum dynamic monopolies and its dual problem of maximum degenerate sets. This is joint work with S. Bessy, C. Brause, L.D. Penso, and D. Rautenbach.

References
Bounds on restricted types of matchings
Maximilian F"urst
Ulm University
Ulm, Germany

Abstract

A matching $M$ in a graph $G$ is called uniquely restricted if there is no other matching in $G$ which covers the same set of vertices, and $M$ is called acyclic if the subgraph induced by the set of vertices of $G$ covered by $M$ is a forest. The maximum cardinalities of a uniquely restricted matching and an acyclic matching are denoted by $\nu_{ur}(G)$ and $\nu_{ac}(G)$, respectively. We present some lower bounds on $\nu_{ur}(G)$ and $\nu_{ac}(G)$ in terms of order, size, and degree. Furthermore, we discuss some possible improvements in graphs without short cycles. This talk is based on joint work with M.A. Henning and D. Rautenbach.

References


Resolution of the Oberwolfach problem
Stefan Glock
University of Birmingham
Birmingham, UK

Abstract

The Oberwolfach problem, posed by Ringel in 1967, asks for a decomposition of $K_{2n+1}$ into edge-disjoint copies of a given 2-factor. We show that this can be achieved for all large $n$. We actually prove a more general result, which allows for decompositions into more general types of factors. In particular, this also resolves the Hamilton–Waterloo problem for large $n$. This is joint work with Felix Joos, Jaehoon Kim, Daniela Kühn and Deryk Osthus.
Planar graphs as L-intersection or L-contact graphs
Daniel Gonçalves
LIRMM, CNRS & Université de Montpellier
Montpellier, France

Abstract
The L-intersection graphs are the graphs that have a representation as intersection graphs of axis parallel L shapes in the plane. We will prove that planar graphs are L-intersection graphs as conjectured by S. Chaplick and T. Ueckerdt in 2013. This result is obtained through a new and simple decomposition technique for 4-connected triangulations. This result also provides a much simpler proof of the known fact that planar graphs are segment intersection graphs.
This is joint work with Lucas Isenmann and Claire Pennarun.

Survey of some results and open problems on the dichromatic number of digraphs
Ararat Harutyunyan
University of Paris-Dauphine
Paris, France

Abstract
The dichromatic number $\overline{\chi}(D)$ of a digraph $D$ is the least number $k$ such that the vertex set of $D$ can be partitioned into $k$ parts each of which induces an acyclic subdigraph. Introduced by Neumann-Lara in 1982 [1], this digraph invariant shares many properties with the usual chromatic number of graphs and can be seen as its natural analog. There has been a renewed interest in this parameter during the last few years. We give a survey of some of the recently obtained results, including results on the list version of the problem. We also discuss some ongoing work and open conjectures.

References
Relating the cut distance and the weak* topology for graphons and hypergraphons

Jan Hladky
Institute of Mathematics of the Czech Academy of Sciences
Prague, Czech Republic

Abstract

The theory of graphons is ultimately connected with the so-called cut norm. We approach the cut norm topology via the weak* topology. We prove that a sequence $W_1, W_2, W_3, \ldots$ of graphons converges in the cut distance if and only if we have equality of the sets of weak* accumulation points and of weak* limit points of all sequences of graphons $W_1', W_2', W_3', \ldots$ that are weakly isomorphic to $W_1, W_2, W_3, \ldots$. We further give a short descriptive set theoretic argument that each sequence of graphons contains a subsequence with the property above. This in particular provides an alternative proof of the theorem of Lovasz and Szegedy about compactness of graphons. This is joint work with Martin Dolezal, Jan Grebik, Israel Rocha and Vaclav Rozhon (arXiv:1806.07368).

We also find ways how to pinpoint cut distance limits in the space of weak* limits as minimizers or maximizers of various graphon parameters. The case of entropy-like parameters is joint work with Martin Dolezal (arXiv:1705.09160), and the general case is joint work with Martin Dolezal, Jan Grebik, Israel Rocha and Vaclav Rozhon.

With Frederik Garbe, Jon Noel, Diana Piguet, Israel Rocha, Maria Saumell we have a similar program for hypergraphons.

A rainbow blow-up lemma

Felix Joos
University of Birmingham
Birmingham, UK

Abstract

We prove a rainbow version of the blow-up lemma of Komlós, Sárközy and Szemerédi for globally bounded edge colourings. In the talk, we shall discuss applications of our blow-up lemma to rainbow embeddings of bounded degree spanning subgraphs and even rainbow decompositions. We will also sketch some ideas of the proof, based on the switching method and the partial resampling algorithm developed by Harris and Srinivasan. This is joint work with Stefan Glock.
The step Sidorenko property and weakly norming graphs
Daniel Král’
University of Warwick
Warwick, UK

Abstract

Sidorenko’s Conjecture asserts that every bipartite graph $H$ has the Sidorenko property, i.e., a quasirandom graph minimizes the density of $H$ among all graphs with the same edge density. We study a stronger property, which requires that a quasirandom multipartite graph minimizes the density of $H$ among all graphs with the same edge densities between its parts; this property is called the step Sidorenko property. We show that many bipartite graphs fail to have the step Sidorenko property. We then use our results to show the existence of a bipartite edge-transitive graph that is not weakly norming, which answers a question of Hatami [Israel J. Math. 175 (2010), 125–150].

The talk is based on joint work with Taísa Martins, Péter Pál Pach and Marcin Wrochna.

Kempe chains and rooted minors
Matthias Kriesell
Technische Universität Ilmenau
Ilmenau, Germany

Abstract

A (minimal) transversal of a partition is a set which contains exactly one member from each member of the partition and nothing else. We study the following problem. Given a transversal $T$ of a proper coloring $\mathcal{C}$ of some graph $G$, is there a partition $\mathcal{H}$ of a subset of $V(G)$ into connected sets such that $T$ is a transversal of $\mathcal{H}$ and such that two sets of $\mathcal{H}$ are adjacent if their corresponding vertices from $T$ are connected by a path using only two colors?

It has been conjectured earlier that for any transversal $T$ of a coloring $\mathcal{C}$ of order $k$ of some graph $G$ such that any pair of color classes induces a connected graph, there exists a partition $\mathcal{H}$ into pairwise disjoint pairwise adjacent sets such that $T$ is a transversal of $\mathcal{H}$ (which would prove Hadwigers conjecture for the class of uniquely colorable graphs); this is open for each $k \geq 5$ and follows from our results for the case that $k = 5$ and the subgraph induced by $T$ is connected.

This is joint work with Samuel Mohr.
On difference graphs and the local dimension of posets
Ryan R. Martin
Iowa State University
Ames, IA, USA

Abstract

The dimension of a partially-ordered set (poset) is the minimum number of linear extensions sufficient to ensure that for every incomparable \( x \) and \( y \), there is one of the extensions that yields \( x < y \). Introduced by Dushnik and Miller, the dimension is a well-studied parameter. However, in any given realization of the dimension of a poset, a given element might not be in many linear extensions.

Ueckerdt introduced the invariant called local dimension which, instead, uses partial linear extensions and which is bounded above by the Dushnik-Miller dimension. For instance, the dimension of the standard example of order \( n \) is \( n/2 \), but the local dimension is only 3.

In this talk, we study the local dimension of show that the maximum local dimension of a poset of order \( n \) is \( \Theta(n/\log n) \), the local dimension of the \( n \)-dimensional Boolean lattice is at least \( \Theta(n/\log n) \) and make progress toward resolving a version of the removable pair conjecture for local dimension. We also connect the computation of local dimension of a poset to the decomposition of the edges of a graph into what are called difference graphs.

This is a joint work with Jinha Kim, Tomáš Masařík, Warren Shull, Heather C. Smith, Andrew Uzzell and Zhiyu Wang.

Unavoidable minors in 2-connected graphs of large pathwidth
Piotr Micek
Jagiellonian University
Kraków, Poland

Abstract

We prove the conjecture of Seymour (1993) that for every apex-forest \( H_1 \) and outerplanar graph \( H_2 \) there is an integer \( p \) such that every 2-connected graph of pathwidth at least \( p \) contains \( H_1 \) or \( H_2 \) as a minor. An independent proof was recently obtained by Dang and Thomas.

Joint work with Tony Huynh, Gwenaël Joret and David R. Wood.
Fifty years of the Ringel and Youngs Theorem
Bojan Mohar
Simon Fraser University and IMFM, Burnaby, Canada and Ljubljana, Slovenia

Abstract

What is the smallest genus of a surface in which the complete graph $K_n$ can be embedded? This question, known as the Heawood problem, was resolved in 1968 by Ringel and Youngs and its solution gave birth to topological graph theory. In the 1990s, Archdeacon and Grable and Rödl and Thomas proved that the genus of random graphs behaves very much like the genus of complete graphs.

The speaker will outline some recent results about genus embeddings of dense graphs building on the work outlined above. The work, which was originally motivated by algorithmic questions, leads to interesting new problems in topological graph theory.

Substantial part of the talk is based on recent joint work with Yifan Jing.
On the maximum number of maximum independent sets
Elena Mohr
Ulm University
Ulm, Germany

Abstract

Zykov’s generalization of Turán’s theorem implies that the number of maximum independent sets of a graph of order \( n \) and independence number \( \alpha \) with \( \alpha \leq n \) is at most \( \left\lceil \frac{n}{\alpha} \right\rceil^{n \mod \alpha} \left\lceil \frac{n}{n \mod \alpha} \right\rceil^{\alpha - (n \mod \alpha)} \). This maximum is attained for the complement of the Turán graph \( T(n, \alpha) \). In this talk we present bounds for the same problem in the class of connected graphs and trees. We also characterize the extremal graphs. This is joint work with D. Rautenbach.

References


Resilience in the random digraph process
Richard Montgomery
University of Birmingham
Birmingham, UK

Abstract
Since the earliest work on random graphs, questions have been asked about which subgraphs are likely to appear. As initiated by Sudakov and Vu around 10 years ago, the limits of our methods here can be explored by studying when a random graph is likely to contain a subgraph resiliently – that is, when it contains a copy of that subgraph despite the removal of many different sets of edges. The goal here is to show that any random graph dense enough to likely contain a copy of a particular subgraph will in fact likely contain a copy of that subgraph resiliently. Directed graph analogues for the resilient containment of Hamilton cycles have been studied by Hefetz, Steeger and Sudakov and Ferber, Nenadov, Noever, Peter and Škorić. I will discuss a resilience version of the hitting-time appearance of Hamilton cycles in the random directed graph process.

Spanning trees in dense directed graphs
Richard Mycroft
University of Birmingham
Birmingham, UK

Abstract
For all $\Delta, \varepsilon > 0$ and sufficiently large $n$, we show that every directed graph $G$ on $n$ vertices with minimum semidegree $\delta^0(G) \geq n/2 + \varepsilon n$ contains every oriented tree $T$ on $n$ vertices with maximum degree $\Delta(T) \leq \Delta$ as a spanning subgraph. I will describe how our proof uses a randomised embedding algorithm to embed most of $T$ in $G$, before completing the embedding by considering distinct cases according to the number of leaves of $T$.

With more work we can embed trees with logarithmic maximum degree (such as random trees) and also embed a range of sparse oriented graphs other than trees.

This is joint work with Tássio Naia.
Triangle-factors in pseudorandom graphs
Rajko Nenadov
ETH Zurich
Zurich, Switzerland

Abstract

We show that if the second eigenvalue $\lambda$ of a $d$-regular graph $G$ on $n \in 3\mathbb{Z}$ vertices is at most $\varepsilon d^2/(n \log n)$, for a small constant $\varepsilon > 0$, then $G$ contains a triangle-factor. The bound on $\lambda$ is at most an $O(\log n)$ factor away from the best possible one: Krivelevich, Sudakov and Szabó, extending a construction of Alon, showed that for every function $d = d(n)$ such that $\Omega(n^{2/3}) \leq d \leq n$ and infinitely many $n \in \mathbb{N}$ there exists a $d$-regular triangle-free graph $G$ with $\Theta(n)$ vertices and $\lambda = \Omega(d^2/n)$. The proof is based on a ‘derandomisation’ of a proof by Krivelevich on the existence of triangle-factors in random graphs.

Algorithmic Graph Theory and Model Theory
Patrice Ossona de Mendez
CAMS (UMR 8557) - CNRS/EHESS and Charles University
Paris, France and Prague, Czech Republic

Abstract

It is a common experience that the sparsity of considered structures allows to find faster more efficient algorithms. This is witnessed by several algorithmic results and meta-theorems related in particular to the nowhere dense/somewhere dense dichotomy. We survey some of the recent highlights and connections with finite model theory. This is a joint work, mainly with Jarik Nešetřil.
Ramsey theory: A higher dimensional view
János Pach
EPFL and Rényi Institute
Lausanne, Switzerland and Budapest, Hungary

Abstract

Many basic problems in extremal graph and hypergraph theory become easier if we restrict our attention to (hyper)graphs whose vertices are points in a Euclidean space and whose edges are defined by simple algebraic or geometric relations. We illustrate this paradigm with the following result. For any integers \( p, m > 2 \), let \( R(p; m) \) denote the smallest positive integer \( R \) such that for any \( m \)-coloring of the edges of \( K_R \), the complete graph on \( R \) vertices, we can find a monochromatic \( K_p \). It is an old problem of Erdős and Schur to decide whether \( R(p; m) = 2^{O(m)} \), for a fixed \( p \). We prove that this is true if we restrict our attention to edge colorings, whose color classes are semi-algebraic sets of bounded complexity. Joint work with Jacob Fox and Andrew Suk.
Unlabelled set partitions with many components
Konstantinos Panagiotou
University of Munich, Institute of Mathematics
Munich, Germany

Abstract

We study combinatorial species $G$ that satisfy the multiset construction in the unlabelled setting, i.e., a structure of such a species is composed of a multiset of structures of an underlying species $C$. Let $G_n$ be drawn at random from $G_n$, the set of structures in $G$ of size $n$, proportional to a compound weight given by the weights of the structures it is composed of. A central parameter in this setting is the distribution of the number of components $\kappa(G_n)$, which is known to converge in distribution. Here we consider the tails of $\kappa(G_n)$ and we determine the asymptotic behaviour of the probability $\Pr(\kappa(G_n) = N)$ for all $N$ such that $n, N$, and $n - N$ tend to infinity, under the rather general assumption that the ordinary (weighted) generating function of $C$ is sub-exponential. As an application we deduce novel enumerative results such as the number of structures in $G_n$ with $N$ components. The proofs are conducted by using (weighted) Boltzmann samplers, by which we reduce the problem of estimating $|G_n|$ to a rather complex property of independent random variables.

This is joint work with Leon Ramzews.
Randomly perturbed graphs
Olaf Parczyk
Technische Universität Ilmenau
Fakultät für Mathematik und Naturwissenschaften
Ilmenau, Germany

Abstract

We study the model of randomly perturbed dense graphs, that is, for any constant $\alpha > 0$, the union of some $n$-vertex graph $G_\alpha$ with minimum degree at least $\alpha n$ and the binomial random graph $G(n, p)$.

We introduce a general approach for studying the appearance of spanning subgraphs in this model. Using this, we can give simpler proofs of several results in the literature concerning the appearance of different spanning subgraphs in this model and obtain new results for bounded degree graphs, powers of Hamilton cycles and universality for bounded degree trees. This addresses two questions of Krivelevich, Kwan, and Sudakov.

This is joint work with Julia Böttcher, Jie Han, Yoshiharu Kohayakawa, Richard Montgomery, and Yury Person.
Tilings in graphs with sublinear independence number

Yanitsa Pehova
University of Warwick
Warwick, UK

Abstract

A classical theorem of Hajnal and Szemerédi states that if a graph $G$ has minimum degree at least $\frac{r-1}{r}|G|$, then it contains a perfect $K_r$-tiling, provided $r$ divides $|G|$. Extremal examples for this result, however, contain large independent sets. Forbidding these extremal structures is likely to decrease the minimum degree condition required to force a $K_r$-tiling, and indeed a result of Balogh, Molla and Sharifzadeh [?] shows that if the independence number of $\alpha(G)$ of $G$ is sublinear, then minimum degree $|G|/2 + \Omega(|G|)$ suffices to force a perfect triangle-tiling. We present an extension of this result for general $K_r$-factors and graphs of sublinear $\ell$-independence number (that is, whose largest $K_\ell$-free subsets have sublinear size). Our proof uses the absorbing method, and our absorber construction can be applied in other settings. For example, we are able to recover a recent result of Balogh, Treglown and Wagner [?] on $H$-factors in dense randomly perturbed graphs. This is joint work with Rajko Nenadov.

References


Packing degenerate graphs
Diana Piguet
Institute of Computer Science, Czech Academy of Sciences
Prague, Czech Republic

Abstract

We say that a family \((H_1, H_2, \ldots, H_k)\) of graphs packs into a graph \(G\), if there are pairwise edges disjoint copies of \(H_i\) in \(G\). In the last few years, there has been a huge development in graph packing.

An \(n\)-vertex graph is \(D\)-degenerate, when there exists an ordering \((v_1, \ldots, v_n)\) of its vertices such that \(|N(v_i) \cap \{v_1, \ldots, v_{i-1}\}| \leq D\) for every \(i \in [n]\). We prove that any family of \(n\)-vertex graphs \(H_i\) of fixed degeneracy and maximum degree at most \(O(n/\log n)\) and whose total amount of edges is at most \((1 - o(1)) \left(\frac{n^2}{2}\right)\) packs in the complete graph \(K_n\). This improves a result of Ferber and Samotij [1] and makes progress on the tree-packing conjecture [2] of Gyárfás from 1976 and the Ringel Conjecture [3] from 1963.

The proof amounts to analysing a very natural randomized algorithm, which embeds most of each graph \(H_i\) in a random way, avoiding to use any edge twice. The left-over part of each graph is then embedded deterministically using a matching argument.

This is joint work with Peter Allen, Julia Böttcher, and Jan Hladký.

References


Stability from symmetrisation arguments
Oleg Pikhurko
University of Warwick
Coventry, UK

Abstract
We present a sufficient condition for the stability property of extremal graph problems that can be solved via Zykov’s symmetrisation. Our criterion is stated in terms of a limit version of the problem. We show that, for example, it applies to the inducibility problem for an arbitrary complete bipartite graph $B$, which asks for the maximum number of induced copies of $B$ in an $n$-vertex graph.

This is joint work with Hong Liu, Maryam Sharifzadeh and Katherine Staden.

A counterexample to Stein’s Equi-$n$-square Conjecture
Alexey Pokrovskiy
ETH Zurich
Zurich, Switzerland

Abstract
In 1975 Stein conjectured that in every $n \times n$ array filled with the numbers $1, \ldots, n$ with every number occurring exactly $n$ times, there is a partial transversal of size $n - 1$ (a partial transversal is a collection of entries which don’t repeat rows, columns, or symbols). This conjecture is important because it generalizes open problems about transversals in Latin squares. In this talk we will present counterexamples to Stein’s Conjecture — there are constructions of such arrays without partial transversals of size $n - \frac{1}{12} \ln n$.

This is joint work with Benny Sudakov.
Induced arboricity
Jonathan Rollin
LG Theoretische Informatik, FernUniversität in Hagen
Hagen, Germany

Abstract

The induced arboricity of a graph $G$ is the smallest integer $k$ s.t. there are $k$ induced forests in $G$ together covering all the edges of $G$. It turns out that this parameter depends on the structure of the graph and not only on its density. This is in contrast to the usual arboricity, where the forests are not necessarily induced.

We discuss some relations between induced arboricity and other graph parameters. This leads to a classification of families of graphs with bounded induced arboricity in terms of chromatic numbers of shallow minors. In particular the induced arboricity is bounded for any family of graphs with bounded expansion. Specifically the largest induced arboricity among all planar graphs lies between 8 and 10.

From an algorithmic point of view we show that deciding whether a graph has induced arboricity at most $k$ is NP-complete for each $k \geq 2$. The problem stays NP-complete for planar graphs in case $2 \leq k \leq 4$.

This is joint work with Maria Axenovich, Philip Dörr, Daniel Gonçalves, and Torsten Ueckerdt.
Gallai Ramsey number for $K_4$ and $K_5$

Ingo Schiermeyer
Technische Universität Bergakademie Freiberg
Freiberg, Germany

Abstract

Given a graph $H$, the $k$-coloured Gallai Ramsey number $gr_k(K_3 : H)$ is defined to be the minimum integer $n$ such that every $k$-colouring (using all $k$ colours) of the complete graph on $n$ vertices contains either a rainbow triangle or a monochromatic copy of $H$. In 2015, Fox, Grinshpun, and Pach [?] conjectured the value of the Gallai Ramsey numbers for complete graphs. The case when $H = K_3$ was actually verified in 1983 by Chung and Graham [?]. We verify this conjecture for the first open case, when $H = K_4$. Finally, we will report about proving the conjecture for the next case, when $H = K_5$. We will show that the validity of the conjecture depends on the exact value of the (unknown) Ramsey number $r(K_5, K_5)$.

This is joint work with Colton Magnant and Akira Saito.

References


On the weak chromatic number of random hypergraphs
Alexander Semenov
Moscow State University, Moscow Institute of Physics and Technology
Moscow, Russia

Abstract

For an integer $j$, a $j$-independent set in a hypergraph $H = (V,E)$ is a subset $W \subset V$ such that for every edge $e \in E : |e \cap W| \leq j$. A $j$-proper coloring of $H = (V,E)$ is a partition of the vertex set $V$ of $H$ into disjoint union of $j$-independent sets, so called colors. The $j$-chromatic number $\chi_j(H)$ of $H$ is the minimal number of colors needed for a $j$-proper coloring of $H$. When $j$ is greater than 1 the corresponding chromatic number usually called weak.

This talk is devoted to weak colorings of $k$-uniform random hypergraph $H(n,k,p)$. We are interested in asymptotic properties of $H(n,k,p)$ to have its $j$-chromatic number equal to some fixed number $r$. By asymptotic properties of $H(n,k,p)$ we consider $n$ as tending to infinity, while $k$ and $r$ are kept constant.

It can be showed that the previously mentioned property of random hypergraph has a sharp threshold. For the case of $(k-1)$-chromatic number upper and lower bounds for that threshold are found. It should be also mentioned that the gap between this bounds is some function $O_k(1)$.

In our previous works we found very tight bounds for the case of $k-j = o(\sqrt{k})$ and $r = 2$. With this work we continue our generalization to the case when $j$ is less than $k-1$, but $r$ is greater than 2. Main result is showed in a theorem below.

**Theorem.** Let $H(n,k,p)$ be a random $k$-uniform hypergraph with $p = cn/(\binom{n}{k})$. For any $r \geq 2$ there exists $k_0 \in \mathbb{N}$, such that if $k > k_0(r)$ and

$$c > \frac{r^{k-1}\ln r}{kr-k+1} - \frac{\ln r}{2} + O\left(\frac{k^{r^2-k}}{kr-k+1}\right)$$

then w.h.p. as $n$ tends to infinity, $\chi_{k-2}(H(n,k,p)) > r$. Otherwise, if

$$c < \frac{r^{k-1}\ln r}{kr-k+1} - \frac{\ln r}{2} + O(k^{-1})$$

then w.h.p. as $n$ tends to infinity, $\chi_{k-2}(H(n,k,p)) \leq r$.

This is joint work with Dmitrii Shabanov. The reported study was funded by RFBR according to the research project 18-302-00001.
Dimension of posets with excluded minors in their cover graphs
Michał Seweryn
Jagiellonian University
Kraków, Poland

Abstract

A fan is a graph obtained from a path by adding an extra vertex adjacent to all vertices on the path. We give a qualitative structure theorem for graphs excluding a fan as a minor. This is inspired by a recent result by Ding that gives an approximate description of graphs excluding $K_{2,n}$ as a minor. Next, we use both characterizations to show that the dimension of a poset is bounded in terms of the size of a largest $K_{2,n}$ or a fan which is a minor of the cover graph. This is a step towards characterization of minor-closed graph classes such that posets with cover graphs from such a class has bounded dimension.
Panchromatic colorings of sparse random hypergraphs
Dmitry Shabanov
Moscow State University, Moscow Institute of Physics and Technology
Moscow, Russia

Abstract

The talk deals with panchromatic colorings of random hypergraphs. Recall that an \( r \)-coloring of the vertex set is said to be panchromatic for a hypergraph \( H = (V, E) \) if every edge meets every color under this coloring. For \( r = 2 \), a panchromatic coloring is just a proper coloring that avoids monochromatic edges in a hypergraph.

We study panchromatic colorings in a setting of random uniform hypergraphs. Let \( H(n, k, p) \) denote a binomial model of a random hypergraph. The main result provides tight bounds for the threshold that \( H(n, k, p) \) admits a panchromatic coloring with \( r \) colors. This threshold corresponds to the sparse case, when the expected number of edges \( p \binom{n}{k} = cn \) is a linear function of \( n \).

We show that for large enough \( k > k_0 \) and \( 4 \leq r < 0.1 \sqrt{k} \), if

\[
c < \frac{\ln r}{r} \left( \frac{r}{r-1} \right)^k - \frac{\ln r}{2} - O \left( k^2 \ln r \left( \frac{r^3}{(r-1)(r+1)^2} \right)^k \right)
\]

then with probability tending to 1 as \( n \to \infty \), the random hypergraph \( H(n, k, \frac{cn}{\binom{n}{k}}) \) admits a panchromatic coloring with \( r \) colors. Moreover, if

\[
c > \frac{\ln r}{r} \left( \frac{r}{r-1} \right)^k - \frac{\ln r}{2} + O \left( \ln r \left( \frac{r(r-2)}{(r-1)^2} \right)^k \right),
\]

then with probability tending to 1 as \( n \to \infty \), the random hypergraph \( H(n, k, \frac{cn}{\binom{n}{k}}) \) does not admit a panchromatic coloring with \( r \) colors.

This is joint work with Nikolay Krokhmal and Dmitry Kravtsov.
The bandwidth theorem for locally dense graphs
Katherine Staden
University of Oxford
Oxford, UK

Abstract

The bandwidth theorem of Böttcher, Schacht and Taraz [?] gives a condition on the minimum degree of an n-vertex graph G that ensures G contains every r-chromatic graph H on n vertices of bounded degree and of bandwidth o(n), thereby proving a conjecture of Bollobás and Komlós. I will discuss a version of the bandwidth theorem for locally dense graphs. This is the statement that every locally dense n-vertex graph G with \( \delta(G) > (1/2 + o(1))n \) contains as a subgraph any given (spanning) H with bounded chromatic number and maximum degree, and sublinear bandwidth.

This is joint work with Andrew Treglown (Birmingham).

References


Rainbow structures, Latin squares and graph decompositions
Benny Sudakov
ETH Zurich
Zurich, Switzerland

Abstract

A subgraph of an edge-coloured graph is called rainbow if all its edges have distinct colours. The study of rainbow subgraphs goes back to the work of Euler on Latin squares. Since then rainbow structures were the focus of extensive research and found applications in design theory and graph decompositions. In this talk we discuss how probabilistic reasoning can be used to attack several old problems in this area. In particular we show that well known conjectures of Ryser, Hahn, Ringel, Graham-Sloane and Brualdi-Hollingsworth hold asymptotically.

Based on joint works with Montgomery, Pokrovskiy and partly with Alon.
Resilience and universality results in random graphs
Anusch Taraz
TU Hamburg
Hamburg, Germany

Abstract

One of the central tasks in the theory of random graphs is to show that a random graph of sufficiently high density contains certain substructures with high probability. In this talk, we focus on the question whether a random graph can do so in a robust manner.

First, we allow an adversary to delete a certain amount of the edges and then we still ask for a copy of a given graph. Second, we would like the random graph to contain all subgraphs from a certain class. We will survey recent progress in this area.

Computing clusters in 3D space
Stéphan Thomassé
Ecole Normale Supérieure de Lyon
Lyon, France

Abstract

I will present an algorithm which takes as input a finite set $X$ of $n$ points of $\mathbb{R}^3$ and computes up to arbitrary precision in poly($n$)-time, a maximum subset of $X$ with diameter at most 1 (cluster). Our algorithm also works for disk graphs of arbitrary radii in the plane and outputs a near optimal collection of pairwise intersecting disks. The main ingredients of the proof are a bit of sampling via VC-dimension, a topological key-lemma, and a large part of graph algorithms.

In stark contrast with dimension 3, we show that finding a maximum cluster in $\mathbb{R}^4$ cannot be 1.001 approximated in $2^{o(n^{0.999})}$ time, unless the Exponential Time Hypothesis fails.

Joint work with Marthe Bonamy, Édouard Bonnet, Nicolas Bousquet and Pierre Charbit
Embeddings in graphs via degree sequence conditions
Andrew Treglown
University of Birmingham
Birmingham, UK

Abstract
Many classical results in extremal graph theory give minimum degree conditions which force a graph $G$ to contain some (spanning) substructure. For example, Dirac’s theorem gives a minimum degree condition that forces a Hamilton cycle. Whilst many such results in the area are best possible in the sense that a lower minimum degree does not guarantee the desired substructure, one can sometimes significantly strengthen these results via degree sequence conditions. Indeed, Chvátal’s theorem generalises Dirac’s theorem by determining all those degree sequences which ensure a Hamilton cycle in a graph.

In this talk I will survey recent progress on such degree sequence results, particularly in the setting of $H$-tilings and powers of Hamilton cycles. The talk will include joint work with Joseph Hyde and Hong Liu; as well as work with Katherine Staden.

Separation for covering numbers
Torsten Ueckerdt
Karlsruhe Institute of Technology
Karlsruhe, Germany

Abstract
Given a graph class $\mathcal{G}$ and a fixed graph $H$, the global $\mathcal{G}$-covering number of $H$ is the smallest number $t$ such that $H$ is the union of $t$ graphs from $\mathcal{G}$. In this talk we introduce three further variants of this parameter: the union, local, and folded $\mathcal{G}$-covering numbers of $H$. We analyze under which conditions on $\mathcal{G}$ and $H$ these parameters can be arbitrarily far apart and under which conditions one $\mathcal{G}$-covering number is bounded by a function of another. These type of questions are closely related to classical concepts such as (induced) Ramsey property of graph classes, or the relative intersection dimension.

This is based on joint work with Thomas Bläsius, Kolja Knauer, Marcel Rademacher, and Peter Stumpf.
Zero $A$-paths and the Erdős-Pósa-property

Arthur Ulmer
Ulm University
Ulm, Germany

Abstract

A class of graphs is said to have the Erdős-Pósa-property if there is a function $f: \mathbb{N} \to \mathbb{N}$ such that in every graph $G$ there are either $k$ disjoint subgraphs that each belong to that class or a set of at most $f(k)$ vertices that intersect all subgraphs of $G$ belonging to that class. Erdős and Pósá showed that cycles have the property (hence the name) and Gallai verified the property for $A$-paths (an $A$-path is a path with both endvertices in $A$ and otherwise disjoint of $A$ for some set of vertices $A$).

As a consequence of a result by Wollan, for any $m \in \mathbb{N}$ also $A$-paths of length not equal to $0$ modulo $m$ have the property, that is $A$-paths whose lengths are not divisible by $m$. We call these paths non-zero $A$-paths.

So what about zero $A$-paths? In the talk I will discuss under which circumstances zero $A$-paths have the Erdős-Pósa-property (and when not).

Almost all string graphs are intersection graphs of plane convex sets

Yelena Yuditsky
Karlsruhe Institute of Technology
Karlsruhe, Germany

Abstract

A string graph is the intersection graph of a family of continuous arcs in the plane. The intersection graph of a family of plane convex sets is a string graph, but not all string graphs can be obtained in this way. We prove the following structure theorem conjectured by Janson and Uzzell: The vertex set of almost all string graphs on $n$ vertices can be partitioned into five cliques such that some pair of them is not connected by any edge ($n \to \infty$). We also show that every graph with the above property is an intersection graph of plane convex sets. As a corollary, we obtain that almost all string graphs on $n$ vertices are intersection graphs of plane convex sets.

This is a joint work with János Pach and Bruce Reed.